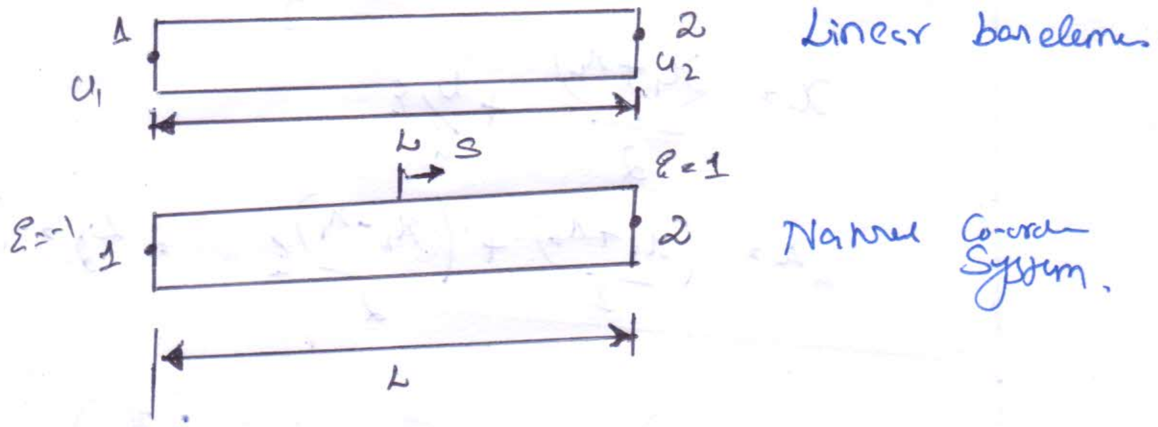
# ONE DIMENSIONAL SHAPE FUNCTIONS FOR

## ISOPARAMETRIC FORMULATION OF THE BAR ELEMENT




Lets consider the bar elem. with 2 nodes.

$u_1, u_2 \to$ respective nodes displacement.

The natural co-ordinate '$\xi$' is attached to the element, with the origin located at the centre of the element.

The '$\xi$' axis need not be parallel to the $x$ axis. this is only for convenience.

We consider the bar element to have two degrees of freedom - axial displacements $u_1$ & $u_2$ at each node associated with the global axis.

When '$\xi$' and '$x$' axes are parallel to each other the '$\xi$' and '$x$' co-ordinates related by

$$x = x_c + \frac{L_i}{2} \varepsilon$$

$x_c \rightarrow$ is the global Coordinate element centroid.

Using the global coordinate $x_1, x_2$.

$$x_c = \frac{(x_1 + x_2)}{2}$$

$$x = \frac{(x_1 + x_2)}{2} + \frac{L}{2}\varepsilon$$

$$x = \frac{(x_1 + x_2)}{2} + \frac{(x_2 - x_1)}{2}\xi \quad ; \; L_g = (x_2 - x_1)$$

$$\left(\frac{x_2 - x_1}{2}\right)\xi = x - \left(\frac{x_1 + x_2}{2}\right)$$

$$\xi = \left(x - \left(\frac{x_1 + x_2}{2}\right)\right)\left(\frac{2}{x_2 - x_1}\right)$$

Since the element has got 2 doF it will have two generalized co-ordinates.

$$\boxed{x = a_1 + a_2 \xi}$$

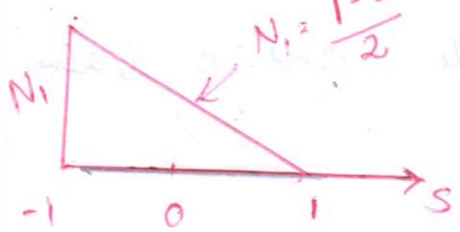
Where $\xi$ is such that $-1 \leq \varepsilon \leq 1$. Solving for the $a_i$ in the terms of $x_1$ and $x_2$ we obtain.

$$x = \frac{1}{2}\left[(1-\varepsilon)x_1 + (\cancel{1} + \xi)x_2\right]$$

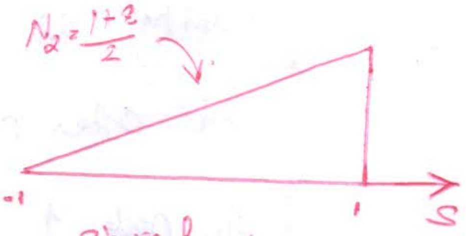
$$\{x\} = \frac{1}{2}\left[(1-\xi) \;\; (1+\xi)\right]\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$$

$$x = N_1 u_1 + N_2 u_2$$

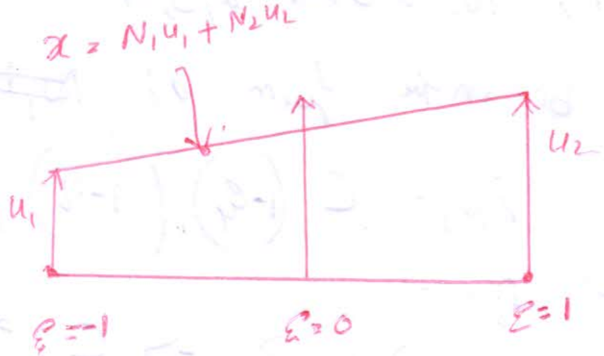
$$\boxed{N_2 = \frac{1+\varepsilon}{2} \quad N_1 = \frac{1-\xi}{2}}$$




Shape functions $N_1$

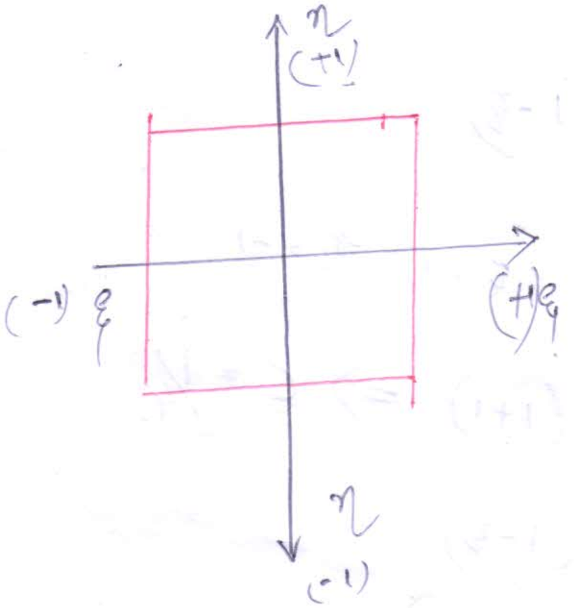


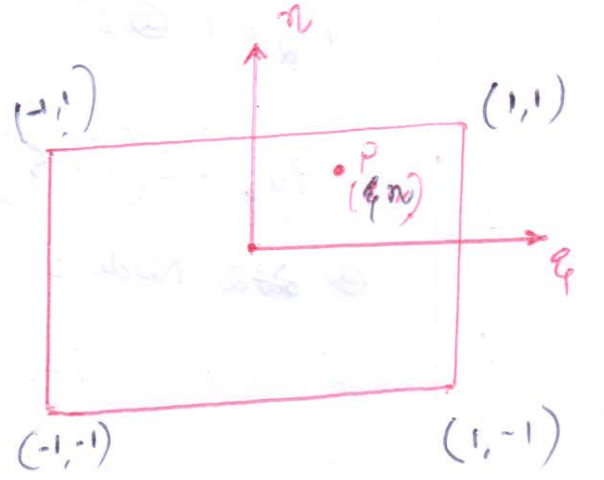

Shape fn $N_2$




Linear displacement field.

Shape Function for 4 noded Rectangular parent element by using Natural Co-ordinate System and Co-ordinate transformation.



We know that the shape function value is unity at its own node and its value is zero at other nodes.

At node 1 $(\xi = -1, \eta = -1)$

at 1 $N_1 = 1$, $N_1 = 0$ at 2, 3, 4

$N_1$ has be in the form of ~~$N_1$~~

$$N_1 = C(1-\xi)(1-\eta), \text{ where C is const.}$$

$\xi = -1, \eta = -1$

$$1 = C(1+1)(1+1) \Rightarrow C = 1/4.$$

$$N_1 = 1/4(1-\xi)(1-\eta)$$

@ node 2 : $(\xi = 1)(\eta = -1)$

$N_2 = 1$ @ 2, $N_2 = 0$ @ 1, 3, 4.

$$N_2 = C(1+\xi)(1-\eta)$$

@ ~~At 2~~ Node 2

$$N_2 = 1, \ \xi = 1, \ \eta = -1.$$

$$1 = C(1+1)(1+1) \Rightarrow C = 1/4.$$

$$N_2 = 1/4(1+\xi)(1-\eta).$$

III$^{ly}$

@ $N_3$ $(\xi = 1, \eta = 1)$. $N_3 = 1$ @ node 3 and $= 0$ at 1, 2, 4.

$$N_3 = C(1+\xi)(1+\eta)$$

$$N_3 = 1/4(1+\xi)(1+\eta)$$

at Node 4 $(\xi = -1, \eta = 1)$

$N_4 = 1$ @ 4
$= 0$ @ 1, 2, 3.

$$N_4 = C(1-\xi)(1+\eta)$$

$$N_4 = \frac{1}{4}(1-\xi)(1+\eta).$$

Consider a point p with co-ordinates $(\xi, \eta)$. If displacement function $u = \begin{Bmatrix} u \\ v \end{Bmatrix}$ represents the displacement components of a point located at $(\xi, \eta)$ then.

$$u = N_1 u_1 + N_2 u_2 + N_3 u_3 + N_4 u_4$$

$$v = N_1 v_1 + N_2 v_2 + N_3 v_3 + N_4 v_4$$

$$u = \begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}$$

In the isoparametric formulation, (ie) for global system, the co-ordinates of the nodal points are $(x_1, y_1)(x_2, y_2)(x_3, y_3)(x_4, y_4)$

In order to get mapping the co-ordinate the point p is defined as.

$$x = N_1 x_1 + N_2 x_2 + N_3 x_3 + x_4 N_4$$

$$y = N_1 y_1 + N_2 y_2 + N_3 y_3 + N_4 y_4$$

The above eqn can be written as,

$$u = \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix} \begin{Bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ x_3 \\ y_3 \\ x_4 \\ y_4 \end{Bmatrix}$$