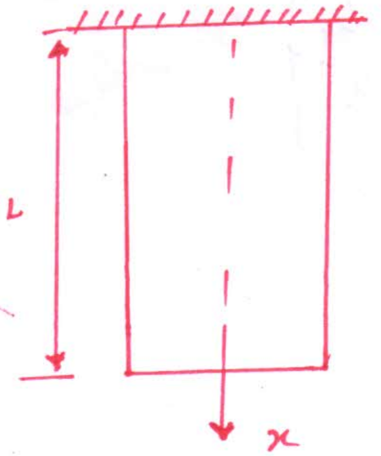
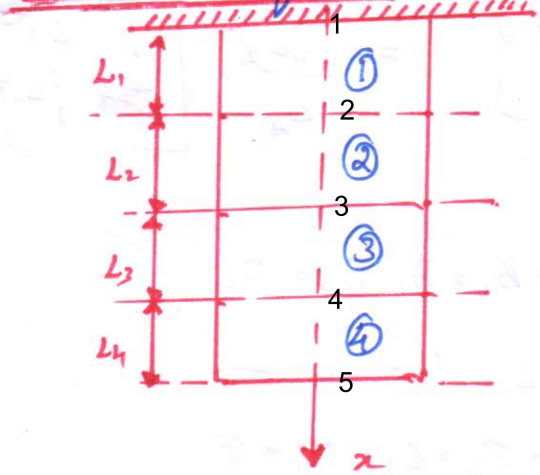
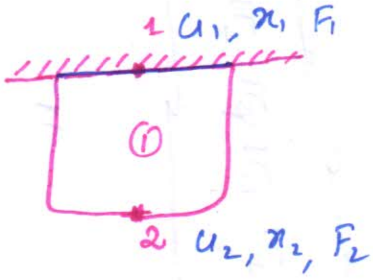
Assembling the Stiffness equations or Global equations.





Element 1 (Nodes 1 & 2)

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{A_1E_1}{L_1} \begin{bmatrix} \overset{a_{11}}{1} & \overset{a_{12}}{-1} \\ \overset{a_{21}}{-1} & \overset{a_{22}}{1} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$



Element . 2. (Nodes (2,3))

$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \frac{A_2E_2}{L_2} \begin{bmatrix} \overset{a_{22}}{1} & \overset{a_{23}}{-1} \\ \overset{a_{32}}{-1} & \overset{a_{33}}{1} \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

Element 3 (Nodes 3 & 4)

$$\begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix} = \frac{A_3E_3}{L_3} \begin{bmatrix} \overset{a_{33}}{1} & \overset{a_{34}}{-1} \\ \overset{a_{43}}{-1} & \overset{a_{44}}{1} \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix}$$

Element: 4 (nodes 4 & 5)

$$\begin{Bmatrix} F_4 \\ F_5 \end{Bmatrix} = \frac{A_4 E_4}{L_4} \begin{bmatrix} a_{44} & a_{45} \\ 1 & -1 \\ a_{54} & a_{55} \\ +1 & 1 \end{bmatrix} \begin{Bmatrix} u_4 \\ u_5 \end{Bmatrix}$$

$A_1 = A_2 = A_3 = A_4 = A.$

$L_1 = L_2 = L_3 = L_4 = L$

$E_1 = E_2 = E_3 = E_4 = E$

Assembly of finite element

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F5 \end{Bmatrix} = \frac{AE}{L} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 1 & -1 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ -1 & 1+1 & -1 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ 0 & -1 & 1+1 & -1 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ 0 & 0 & -1 & 1+1 & -1 \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix}$$

$$= \frac{AE}{L} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix}$$

$[K]_{global}$

Stress, Strain, Displacement and Loading:

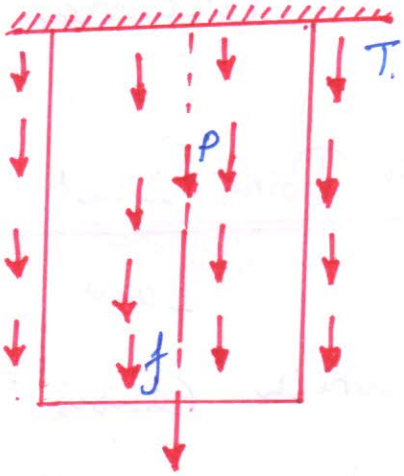
⇒ in 1D problems, stress(σ), strain(e), displacement(u) & loading depends only on the variable x.

So the vectors u, σ & e can be written as

$u = u(x)$

$\sigma = \sigma(x)$

$e = e(x)$.



stress strain relation

$$\sigma = E e$$

strain displacement relation.

$$e = \frac{du}{dx}$$

The differential volume can be written as

$$dV = A.dx.$$

$V = A \times l$.
Vol = Area × length.

Three types of loading acts on the body

(i) Body force (f)

(ii) Traction force (T)

(iii) Point Load (P)

(i) Body force (f)

It's the distributed force acting on every elemental volume of the body.

Unit → force / volume (eg) Self weight of body due to gravity.

(ii) Traction force (T)

Its a distributed force acting on the surface of the body.

Unit: force per unit area. (for 1D, unit/length)

(eg) frictional resistance, viscous drag, etc

(iii) Point Load (P)

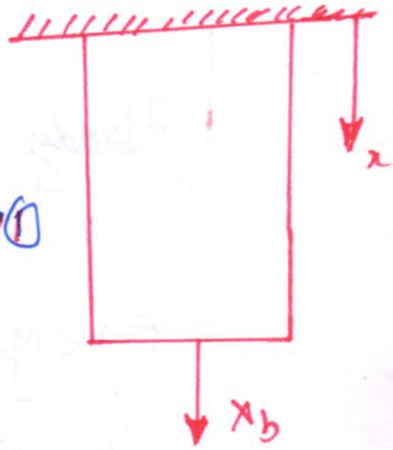
Load acting at a particular point which causes displacement.

The Load or Force Vector $\{F\}$.

→ Consider a vertically hanging bar of length l, uniform cross section A, density ρ and youngs modulus E. The self weight of bar is X_b.

The elemental nodal force vector is given by $\{F_e\} = \int [N]^T X_b$. → ①

$X_b = \rho \cdot A \cdot dx$. → ②



We know that for 1D. $u = N_1 u_1 + N_2 u_2$

$N_1 = \frac{l-x}{l}$, $N_2 = \frac{x}{l}$.

$$[N] = \left[\frac{1-x}{l}, \ l/x\right] \Rightarrow [N]^T = \begin{Bmatrix} \frac{l-x}{l} \\ l/x \end{Bmatrix} \rightarrow (2)$$

Substin the Valu of (2) & (3) in (1) we get

$$\{F_e\} = \int_0^l \begin{Bmatrix} \frac{l-x}{l} \\ \frac{x}{l} \end{Bmatrix} \rho A . dx = \rho A \int_0^l \begin{Bmatrix} \frac{l-x}{l} \\ \frac{x}{l} \end{Bmatrix} dx$$

$$= \rho A \int_0^l \begin{Bmatrix} 1 - \frac{x}{l} \\ \frac{x}{l} \end{Bmatrix} . dx$$

$$= \rho A \begin{Bmatrix} x - \frac{x^2}{2l} \\ \frac{x^2}{2l} \end{Bmatrix}_0^l = \rho A \begin{Bmatrix} l - \frac{l^2}{2l} \\ l/2 \end{Bmatrix}$$

$$\{F_e\} = \rho A l \begin{Bmatrix} 1/2 \\ 1/2 \end{Bmatrix}$$

$$\{F_e\} = \frac{\rho A l}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$