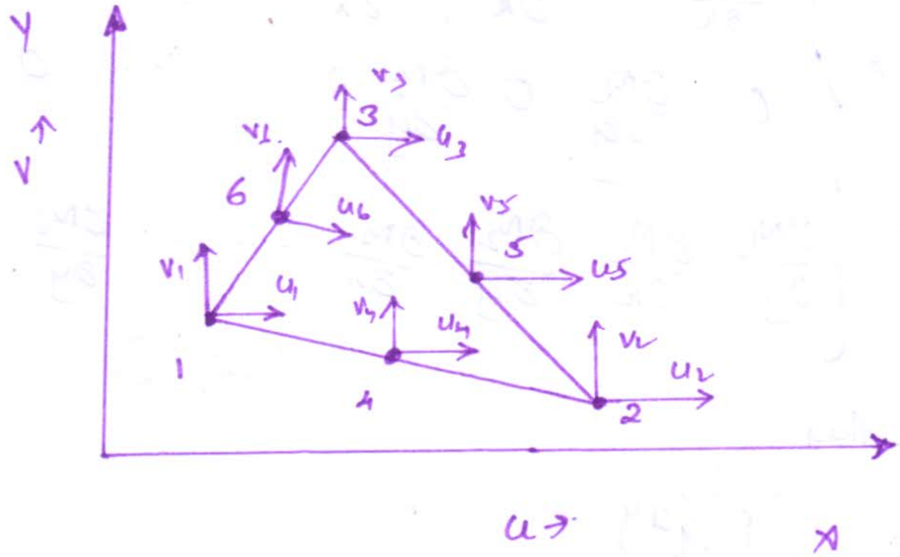
LST (Linear Strain Triangular) ELEMENT



- A Six Noded triangular element is known as LST Element
- 12 unknown D.O.F. The displacement function of the element is quadratic instead of linear as in the CST.

$$U\{u, v\} = \{u_1, v_1, u_2, v_2, u_3, v_3, u_4, v_4, u_5, v_5, u_6, v_6\}^T$$

$$\left.\begin{aligned} u &= u_1N_1 + N_2u_2 + N_3u_3 + N_4u_4 + N_5u_5 + N_6u_6 \\ v &= v_1N_1 + N_2v_2 + N_3v_3 + N_4v_4 + N_5v_5 + N_6v_6 \end{aligned}\right\} \rightarrow (1)$$

$$u = \sum_{i=1}^{6} N_i u_i \qquad v = \sum_{i=1}^{6} N_i v_i$$

We know that the Shape functions.

$$\left.\begin{aligned} N_1 &= (2L_1-1)L_1 & N_4 &= 4L_1L_2 \\ N_2 &= L_2(2L_2-1) & N_5 &= 4L_2L_3 \\ N_3 &= L_3(2L_3-1) & N_6 &= 4L_1L_3 \end{aligned}\right\} \rightarrow (2)$$

$$\begin{Bmatrix} e_{xx} \\ e_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \frac{\partial N_3}{\partial x} & 0 & \cdots & \frac{\partial N_6}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & 0 & \cdots & \cdots & 0 & \frac{\partial N_6}{\partial y} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \cdots & \cdots & \cdots & \frac{\partial N_6}{\partial y} & \frac{\partial N_6}{\partial x} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ \vdots \\ u_6 \\ v_6 \end{Bmatrix} \rightarrow (3)$$

we know that

$$\{e\} = [B]\{u\}.$$

The informa needed to express deri of Shape funct. w.r.t. global Co-ordinates x + y is given below usi Chain rule.

$$\left.\begin{aligned} \frac{\partial N_1}{\partial x} &= \frac{\partial N_1}{\partial L_1}\cdot\frac{\partial L_1}{\partial x} + \frac{\partial N_1}{\partial L_2}\cdot\frac{\partial L_2}{\partial x} + \frac{\partial N_1}{\partial L_3}\cdot\frac{\partial L_3}{\partial x} \\ \frac{\partial N_1}{\partial y} &= \frac{\partial N_1}{\partial L_1}\cdot\frac{\partial L_1}{\partial y} + \frac{\partial N_1}{\partial L_2}\cdot\frac{\partial L_2}{\partial y} + \frac{\partial N_3}{\partial L_3}\cdot\frac{\partial L_3}{\partial y} \end{aligned}\right\} (4)$$

area Co-ordinates are expressed as

$$\left.\begin{aligned} L_i &= a_i + b_i x + c_i y \\ a_i &= (x_j y_k - y_j x_k)/2A \qquad c_i = \frac{(x_k - x_j)}{2A} \\ b_i &= \frac{(y_j - y_k)}{2A} \end{aligned}\right\} \rightarrow (5)$$

(Note the $2A$ is include in the defin of coeffn.

$$\left.\begin{aligned} &\frac{\partial L_1}{\partial x} = b_1 \qquad \frac{\partial L_2}{\partial x} = b_2 \qquad \frac{\partial L_3}{\partial x} = b_3 \\ &\frac{\partial L_1}{\partial y} = c_1 \qquad \frac{\partial L_2}{\partial y} = c_2 \qquad \frac{\partial L_3}{\partial y} = c_3. \end{aligned}\right\} \quad (6)$$

Using eqn. (2), (4) + (6).

We know that

$$\cancel{\frac{\partial N_1}{\partial x} = \frac{\partial L_1}{\partial x}\cdot(2L_1 - 1)}$$

$$\frac{\partial N_1}{\partial x} = \frac{\partial N_1}{\partial L_1}\cdot\frac{\partial L_1}{\partial x} + \frac{\partial N_1}{\partial L_2}\cdot\frac{\partial L_2}{\partial x} + \frac{\partial N_1}{\partial L_3}\cdot\frac{\partial L_3}{\partial x}$$

$$= \frac{\partial}{\partial L_1} L_1(2L_1 - 1)\cdot b_1 + 0 + 0$$

$$= \frac{\partial}{\partial L_1}(2L_1^2 - L_1)\, b_1 = \underline{(4L_1 - 1)\, b_1}$$

$$\boxed{\frac{\partial N_1}{\partial x} = b_1(4L_1 - 1)}$$

IIIly.

$$\left.\begin{aligned} &\frac{\partial N_1}{\partial x} = b_1(4L_1 - 1) \qquad &&\frac{\partial N_1}{\partial y} = c_1(4L_1 - 1) \\ &\frac{\partial N_2}{\partial x} = b_2(4L_2 - 1) \qquad &&\frac{\partial N_2}{\partial y} = c_2(4L_2 - 1) \\ &\frac{\partial N_3}{\partial x} = b_3(4L_3 - 1) \qquad &&\frac{\partial N_3}{\partial y} = c_3(4L_3 - 1) \end{aligned}\right\}$$

$$\frac{\partial N_4}{\partial x} = 4(L_2 b_1 + L_1 b_2), \quad \frac{\partial N_4}{\partial y} = 4(L_2 c_1 + L_1 c_2)$$

$$\frac{\partial N_5}{\partial x} = 4(L_3 b_2 + L_2 b_3), \quad \frac{\partial N_5}{\partial y} = 4(L_3 c_2 + L_2 c_3)$$

$$\frac{\partial N_6}{\partial x} = 4(L_1 b_3 + L_3 b_1), \quad \frac{\partial N_6}{\partial y} = 4(L_1 c_3 + L_3 c_1)$$

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$$\{\sigma\} = [D]\{e\}$$

Stran energy in the element is $U_e = \frac{1}{2}\int_V \{e\}^T\{\sigma\} \cdot dV$.

$$U_e = \frac{1}{2}\{u\}^T \left[\int_V [B]^T[D][B] \cdot dV\right]\{u\}$$

$$[K] = \left[\int_V [B]^T[D][B] \cdot dV\right.$$