Numerical Integration (Gaussian Quadrature) and application to plane Stress problems.

The Gauss quadrature is one of the Numerical integration methods to Calculate the definite integrals. In FEA, Gauss-quadrature method is mostly preffed. In this method, the numerical integration is achieved by the following expression.

$$\int_{-1}^{1} f(x)\cdot dx = \sum_{i=1}^{n} W_i f(x_i)$$

Where, W_i is weight function.

$f(x_i)$ is value of the function at pre-determined sampling points.

Function $f(x_i)$ is Calculated at several sampling points (ie) $n = 1, 2, 3$ and each value of $f(x)$ is multiplied by weight function W_i.

Finally all the terms are added, it gives the value of integral.

Table shows the location of Gauss Sampling Points $f(x_i)$ and Corresponding weights for w_i, for different number of points (n).

$$\int_{-1}^{1} f(x)\,dx = \sum_{i=1}^{n} w_i f(x_i)$$

Number of points, n	Location, x_i	Corresponding weights, w_i
1	$x_1 = 0.000.....$	2.000
2	$x_1, x_2 = \pm\sqrt{\frac{1}{3}} = \pm 0.577350269189$	1.000
3	$x_1, x_3 = \pm\sqrt{\frac{3}{5}} = \pm 0.774596669241$ $x_2 = 0.000$	$\frac{5}{9} = 0.555555.....$ $\frac{8}{9} = 0.888888.....$
4	$x_1, x_4 = \pm 0.8611363116$ $x_2, x_3 = \pm 0.3399810436$	0.3478548451 0.6521451549

Evaluat $\int_{-1}^{1} (x^4+x^2)\,dx$ by applying 3 point

Gaussian quadrature.

$$I = \int_{-1}^{1} (x^4+x^2)\,dx$$

$$f(x) = x^4+x^2$$

From the table.

$x_1 = \sqrt{3/5}$, $x_3 = -\sqrt{3/5}$, $x_2 = 0$.

$W_1 = 5/9$, $W_2 = 8/9$, $W_3 = 5/9$

$f(x_3) = x^4+x^2$

$f(x_1) (0.7746)^4 + (0.7746)^2$

$f(x_1) = 0.96$

$W_1 f(x_1 = 0.5555 \times 0.96 = 0.5333$

$f(x_2) = (x_2^4) + (x_2^2) = 0$

$W_2 f(x_2) = 0$

$f(x_3) = (x_3)^4 + (x_3)^2 = (-0.7746)^4 + (-0.7746)^2$

$f(x_3) = 0.96$

$W_3 f(x_3) = 0.5555 \times 0.96 = 0.5333$

$W_1 f(x_1) + W_2 f(x_2) + W_3 f(x_3) = 0.5333 + 0 + 0.5333$

$\int_{-1}^{1} (x^4+x^2)\,dx = 1.0666$

Evaluate the integral by using Gauss quadrature $\int_{-1}^{1} x^2 dx$.

$2n-1$ = order of polynomial

$f(x) = x^2$ power of 2

Here n

$2n-1 = 2$.

$2n = 3$

$n = 3/2 = 1.5$

$x_1 = +\sqrt{1/3}$ $\quad w_1 = 1$

$x_2 = -\sqrt{1/3}$ $\quad w_2 = 1$

$f(x_1) = 0.333$. $\quad w_1(f_1) = 0.333$

$f(x_2) = 0.333$ $\quad w_2(f_2) = 0.333$.

Adding ① & ②

$w_i f(x_i) + w_2 f(x_2) = 0.333 + 0.333 = 0.666$

$$\int_{-1}^{1} x^2 dx = 0.666$$