M: 2

Numericals.

1) Calculate the Value of $\int_0^l L_1 L_2 \, dx$

Soln

We know

$$\int_A (L_1)^{\alpha} (L_2)^{\beta} dx = \frac{\alpha! \, \beta!}{(\alpha+\beta+1)!} \times l$$

Comparing $\alpha = 1$, $\beta = 1$

$$\frac{\alpha! \, \beta!}{(\alpha+\beta+1)!} l = \frac{1! \, 1!}{3!} l = \frac{L}{6}$$

② Calculate the Value of $\int_A L_1 L_2^2 L_3^3 \, dA$

Soln

$$\int_A (L_1)^{\alpha} (L_2)^{\beta} (L_3)^{\gamma} dA = \frac{\alpha! \, \beta! \, \gamma!}{(\alpha+\beta+\gamma+2)!} 2A.$$

$\alpha = 1$, $\beta = 2$, $\gamma = 3$.

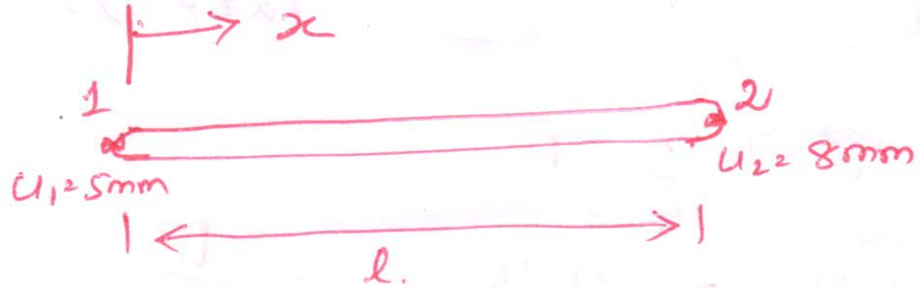
$$= \frac{1! \times 2! \times 3!}{(1+2+3+2)!} 2A$$

$$= \frac{A}{1680}.$$

AW

① Determine the value of $\int_0^l L_1^3 \cdot dx$.

③ A two noded truss element is shown in figure. The nodal displacement are $u_1 = 5mm$ & $u_2 = 8mm$. Calculate the displacement at $x = l/4$, $l/3$ & $l/2$



We know that: $u_1 = 5mm$, $u_2 = 8mm$

$$u = N_1 u_1 + N_2 u_2, \quad N_1 = \frac{l-x}{l}, \quad N_2 = \frac{x}{l}.$$

$$u = \left(\frac{l-x}{l}\right) u_1 + \left(\frac{x}{l}\right) u_2 \rightarrow ①$$

to find displacement at $x = l/4$.

$$u = \left(\frac{l - l/4}{l}\right) 5 + \left(\frac{l/4}{l}\right) 8mm$$

@ $x = l/4$, $u = \cancel{6mm}$ 5.75mm

to find disp @ $x = l/3$

$$u = \left(\frac{l - l/3}{l}\right) 5 + \left(\frac{l/3}{l}\right) 8$$

$u = 6mm$ @ $x = l/3$.

@ $x = l/2$

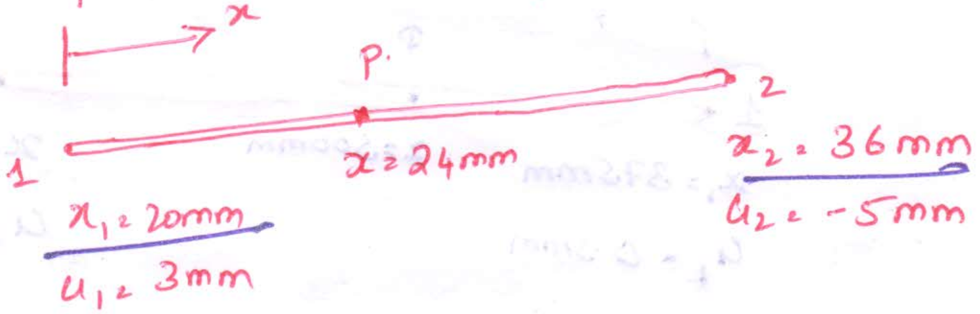
$$u = \left(\frac{l - l/2}{l}\right) u_1 + \cancel{85} + \left(\frac{l/2}{l}\right) u_2$$

$u = 6.5\,mm$ @ $x = l/2$

4 A 1D bar is shown. Calculate the following

(i) Shape function N_1 & N_2 at point P.

(ii) if $u_1 = 3\,mm$ & $u_2 = -5\,mm$. Calculate the displacement u at point P.



Soln

W.K.T. $l = x_2 - x_1 = 16\,mm$

the distance b/n ~~point~~ node 1 and point P is 4mm.
($x = 24 - 20\,mm$)

$u = N_1 u_1 + N_2 u_2$.

$N_1 = \frac{l - x}{l} = \frac{16 - 4}{16} = 0.75\,mm$

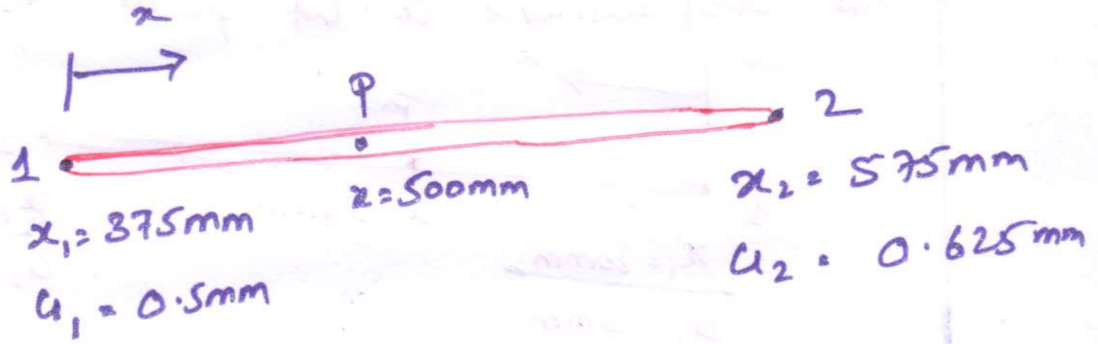
$N_2 = \frac{x}{l} = 4/16 = 0.25\,mm$

$u = 0.75 \times 3 + 0.25(-5)$

$u = 1\,mm$

(3) Consider a bar as shown in figure. Cross sectional area of the bar is $750\,mm^2$ and Young's modulus is $2 \times 10^5\ N/mm^2$. If $u_1 = 0.5mm$ and $u_2 = 0.625mm$. Calculate the following.

(i) Displacement at point, P. (ii) Strain, e (iii) Stress, σ
(iv) Element stiffness matrix $[k]$ (v) Strain energy, U



Soln:

Length of bar $= x_2 - x_1 = 575 - 375 = 200\,mm.$

Distance of P from node 1, $x = 500 - 375 = 125\,mm.$

Displacement $u = N_1 u_1 + N_2 u_2$

$N_1 = \frac{l-x}{l} = N_1 = \frac{200-125}{200} = 0.375$

$N_2 = \frac{x}{l} = \frac{125}{200} = 0.625$

(i) Displacement

$$u = N_1 u_1 + N_2 u_2$$

$$= 0.375\,(0.5) + 0.625\,(0.625)$$

$$u = 0.5781 \text{ mm.}$$

(ii) Strain(e) $= [B]\{u^*\}$

$$[B] = \begin{bmatrix} -\frac{1}{l} & \frac{1}{l} \end{bmatrix}, \quad \{u^*\} = \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{200}, & \frac{1}{200} \end{bmatrix} \begin{Bmatrix} 0.5 \\ 0.625 \end{Bmatrix}$$

$$e = 6.25 \times 10^{-4}.$$

(iii) Stress, $\sigma = E \times e$

$$= 2 \times 10^5 \times 6.25 \times 10^{-4}$$

$$= 125 \text{ N/mm}^2$$

(iv) Element Stiffness matrix $[k]$

$$[k] = \frac{AE}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{750 \times 2 \times 10^5}{200} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[k] = 7.5 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

(5) Strain energy $U = \frac{1}{2}\{u^*\}^T [K] \{u^*\}$

$= \frac{1}{2} [u_1, u_2) \times 7.5 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$

$= \frac{1}{2} [0.5, 0.625] \times 7.5 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0.5 \\ 0.625 \end{Bmatrix}$

$= \frac{7.5 \times 10^5}{2} [0.5, 0.625] \begin{bmatrix} 0.5 - 0.625 \\ -0.5 + 0.625 \end{bmatrix}$

$= \frac{7.5 \times 10^5}{2} [0.5, 0.625) \begin{Bmatrix} -0.125 \\ 0.625 \end{Bmatrix}$

$U = 5859.37$ N-mm

6) A Steel bar of length 800 mm is subjected to an axial load of 3 kN as shown in figure. Find the elongation of the bar, neglect self weight.

Take $E = 2 \times 10^5$ N/mm^2, $A = 300$ mm^2