Serendipity Elements:

Serendipity Shape function.

Shape functions for nodes only on sides of a parent square element are known as serendipity elements.

(i) Four noded serendipity element.

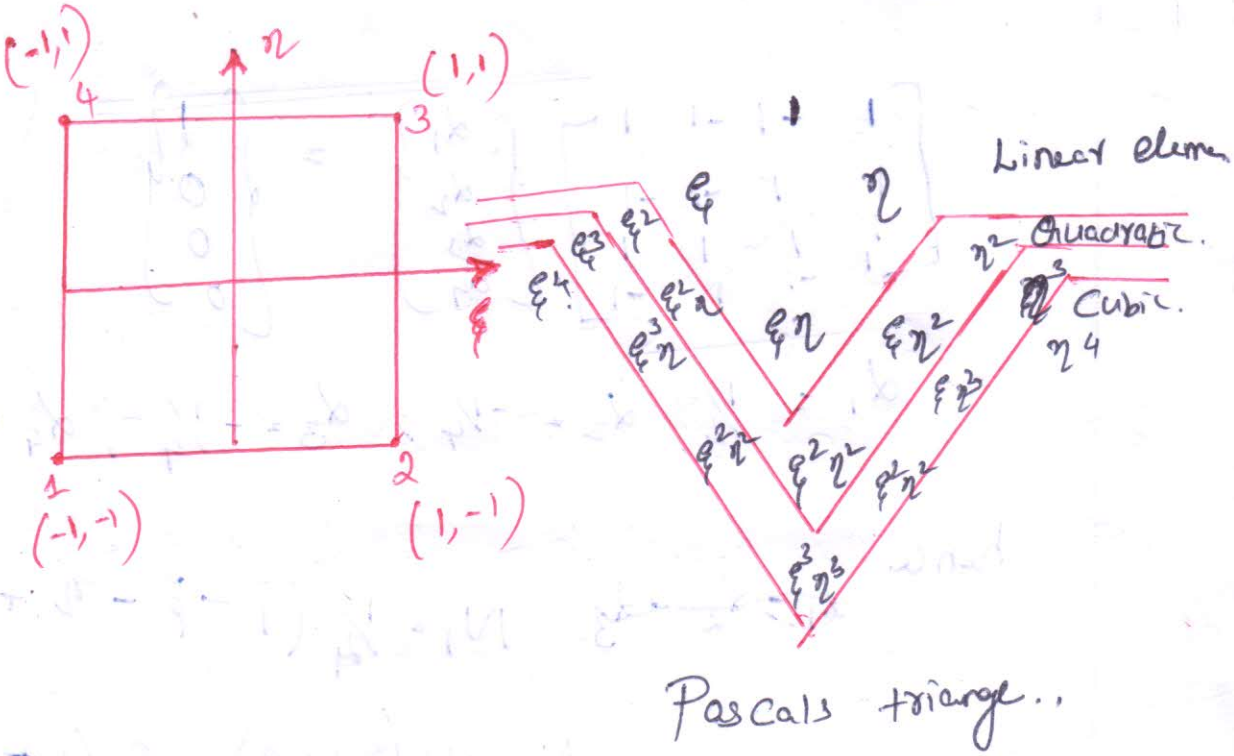

figure shows four noded square element in the region $(-1 \le \xi \le 1)$ & $(-1 \le \eta \le 1)$

Shape function N_1 has unit value at node 1 $(\xi = \eta = -1)$ and zero values at other nodes.

Since function values are known as four nodes, a polynomial involving four constants is used to express N_1; Lowest order terms in ξ & η are used thus.

$$N_1 = \alpha_1 + \alpha_2 \xi + \alpha_3 \eta + \alpha_4 \xi \eta.$$

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Substn the boundary conditn.

$(-1\ -1)$
$(1\ -1)$
$(1, 1)$
$(-1, 1)$

$$\begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$\alpha_1 = 1/4, \ \alpha_2 = -1/4, \ \alpha_3 = -1/4, \ \alpha_4 = 1/4$

hence $\cancel{\alpha_1 = \alpha_2 = \alpha_3}$ $N_1 = \frac{1}{4}(1 - \xi - \eta + \xi\eta)$

$\cancel{N_1 = \frac{1}{4}((1-\xi) + \eta(1+}$

$N_1 = \frac{1}{4}\left[1(1-\xi) - \eta(1+\xi)\right]$

$N_1 = \frac{1}{4}\left[(1-\eta)(1-\xi)\right]$

(or)

above inclvdes soln for 4×4 matrix or in simple we can use

Nodes 2 & 3 are on line $(1-\xi) = 0$ and nodes 3 & 4 are on line $(1-\eta) = 0$

$N_1 = C(1-\xi)(1-\eta)$

To get unit value at node 1 $(\xi = \eta = -1)$

$1 = C(1+1)(1+1)$

$C = 1/4.$

$N_1 = 1/4\,(1-\xi)(1-\eta)$

$$N_2 = \frac{1}{4}(1+\xi)(1-\eta)$$

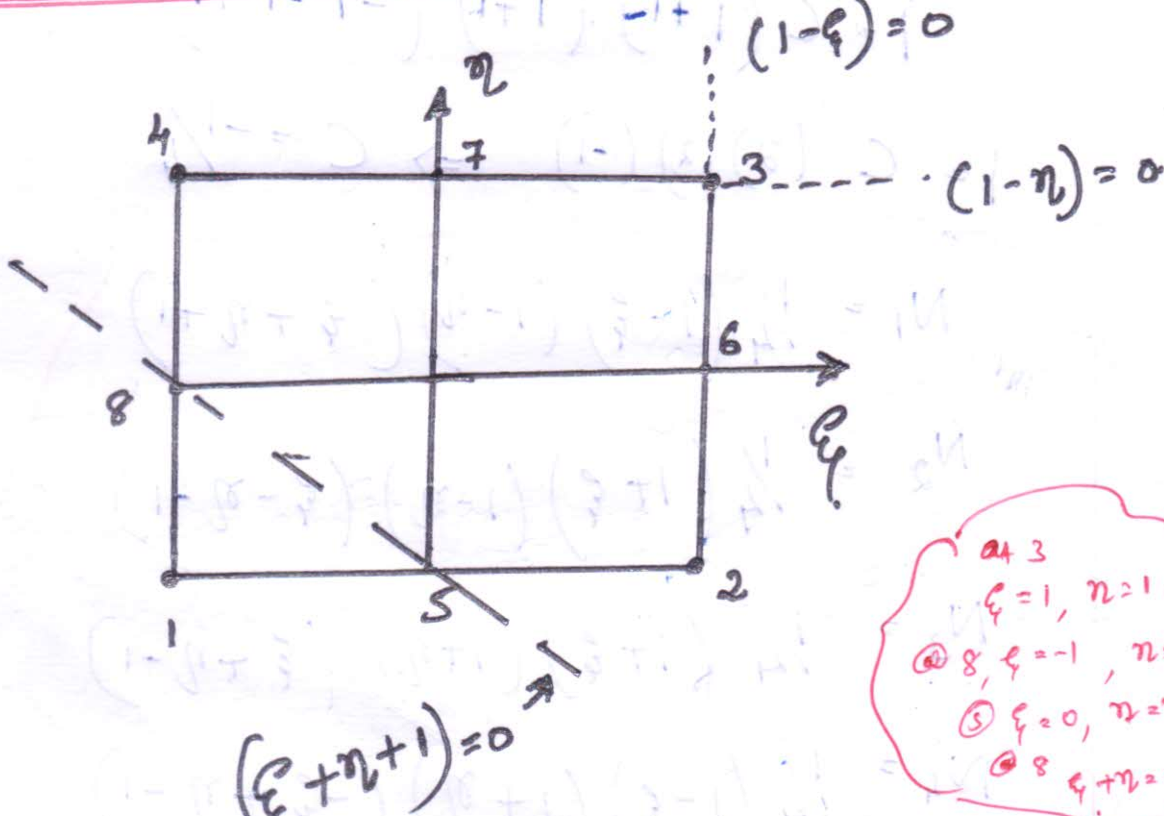
$$N_3 = \frac{1}{4}(1+\xi)(1+\eta)$$

$$N_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

All Shape functions are written using a Concise expression as

$$N_i = \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i) \quad (\text{for } i = 1 \text{ to } 4.)$$

Eight noded serendipity elements.



@ 3
$\xi = 1, \eta = 1.$
@ 8, $\xi = -1, \eta = 0.$
⑤ $\xi = 0, \eta = -1.$
@ 8 $\xi + \eta = -1$

The figure shows a eight

noded serendipity elements. Nodes 2,6,3 are on line $(1-\xi)=0$, and nodes 4,7,3 are on line $(1-\eta)=0$

hence it can be written as.

$$N_1 = C(1-\xi)(1-\eta)(\xi+\eta+1)$$

these 3 expn will give zero value at N_1 at all nodes except at node 1. To get unit value of N_1 at node 1 ($\xi=\eta=-1$) we use.

$$1 = C(1+1)(1+1)(-1-1+1)$$

$$1 = C(2)(2)(-1) \Rightarrow C = -1/4$$

$$N_1 = 1/4(1-\xi)(1-\eta)(\xi+\eta+1)$$

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$$N_2 = 1/4(1+\xi)(1-\eta)(\xi-\eta-1)$$

$$N_3 = 1/4(1+\xi)(1+\eta)(\xi+\eta-1)$$

$$N_4 = 1/4(1-\xi)(1+\eta)(-\xi+\eta-1)$$

Shape function N_6 has zero value at all nodes enept at node 5. Thus, arrive by wrtn

$$\cancel{N_5(}\ N_5 = C(1-\xi)(1+\xi)(1-\eta)$$

To get unit value of N_5 at node 5 $(\xi = 0, \eta = -1)$

$$1 = C(1)(1)(2) = 2C \Rightarrow C = 1/2$$

$$N_5 = 1/2\,(1-\xi^2)(1-\eta)$$

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$$N_6 = 1/2\,(1-\eta^2)(1+\xi)$$

$$N_7 = 1/2\,(1-\xi^2)(1+\eta)$$

$$N_8 = 1/2\,(1-\eta^2)(1-\xi)$$

for midel nodes.

5 & 7 $\quad N_i = 1/2\,(1-\xi^2)(1+\eta\eta_i)$

6 & 8 $\quad N_i = 1/2\,(1-\eta^2)(1+\xi\xi_i)$