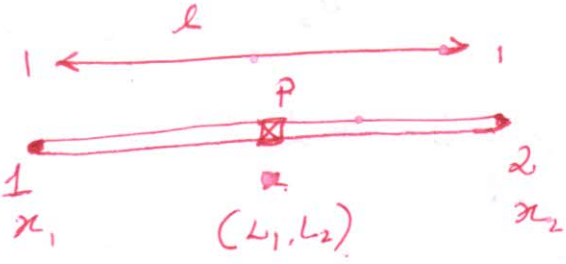
MODULE: 2

Natural Co-ordinates in One Dimension. (L_1 & L_2)



Consider a two noded element as shown in figure. At any point p inside the ~~and~~ line element is identified by two natural Co-ordinates L_1 and L_2 and the Cartesian Co-ordinates x: ~~L_1 and L_2~~ Node 1 & Node 2 have the Cartesian Co-ordinates x_1 & x_2 respectively.

We know that
The total weightage of Natural co-ordinates at any point is unity

$$L_1 + L_2 = 1 \longrightarrow (1)$$

At any point x within the element can be expressed as a linear combination of the nodal Co-ordinates of nodes 1 and 2 as

$$L_1 x_1 + L_2 x_2 = x \longrightarrow (2)$$

arranging ① & ② in matrix form.

$$\begin{bmatrix} 1 & 1 \\ x_1 & x_2 \end{bmatrix} \begin{Bmatrix} L_1 \\ L_2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ x \end{Bmatrix}$$

We need to find L_1 & L_2

$$\begin{bmatrix} 1 & 1 \\ x_1 & x_2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ x_1 & x_2 \end{bmatrix} \begin{Bmatrix} L_1 \\ L_2 \end{Bmatrix} = \begin{bmatrix} 1 & 1 \\ x_1 & x_2 \end{bmatrix}^{-1} \begin{Bmatrix} 1 \\ x \end{Bmatrix}$$

$\boxed{\text{Note: } A^{-1}A = AA^{-1} = I}$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} L_1 \\ L_2 \end{Bmatrix} = \begin{bmatrix} 1 & 1 \\ x_1 & x_2 \end{bmatrix}^{-1} \begin{Bmatrix} 1 \\ x \end{Bmatrix}$$

$$A^{-1} = \frac{1}{|A|} C^T$$

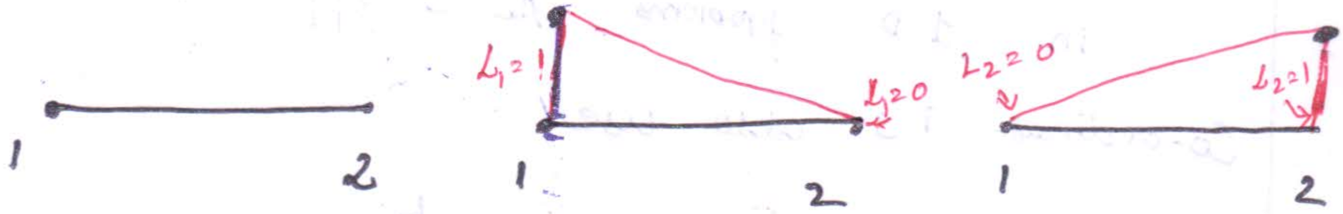
$$\begin{Bmatrix} L_1 \\ L_2 \end{Bmatrix} = \frac{1}{(x_2 - x_1)} \begin{bmatrix} x_2 & -1 \\ -x_1 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ x \end{Bmatrix}$$

$(x_2 - x_1 = l$ length of elem$)$

$$= \frac{1}{l} \begin{bmatrix} x_2 - x \\ -x_1 + x \end{bmatrix} = \frac{1}{l} \begin{bmatrix} x_2 - x \\ x - x_1 \end{bmatrix}$$

$$\begin{Bmatrix} L_1 \\ L_2 \end{Bmatrix} = \begin{Bmatrix} \dfrac{x_2 - x}{l} \\ \dfrac{x - x_1}{l} \end{Bmatrix}$$

The Variation of L_1 and L_2



Element with node nos.

Variation of L_1

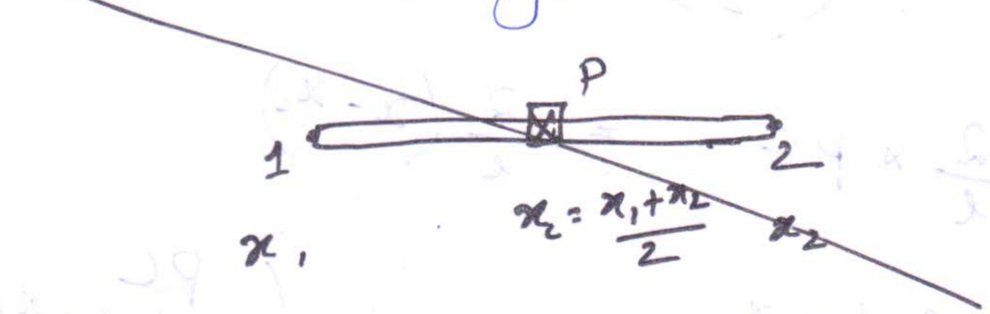
Variation of L_2

The integration of polinomial terms in natural co-ordinate can be performed by a formula.

$$\int_{x_1}^{x_2} (L_1)^{\alpha} (L_2)^{\beta} \cdot dx = \frac{\alpha!\,\beta!}{(\alpha+\beta+1)!} \times l_x.$$

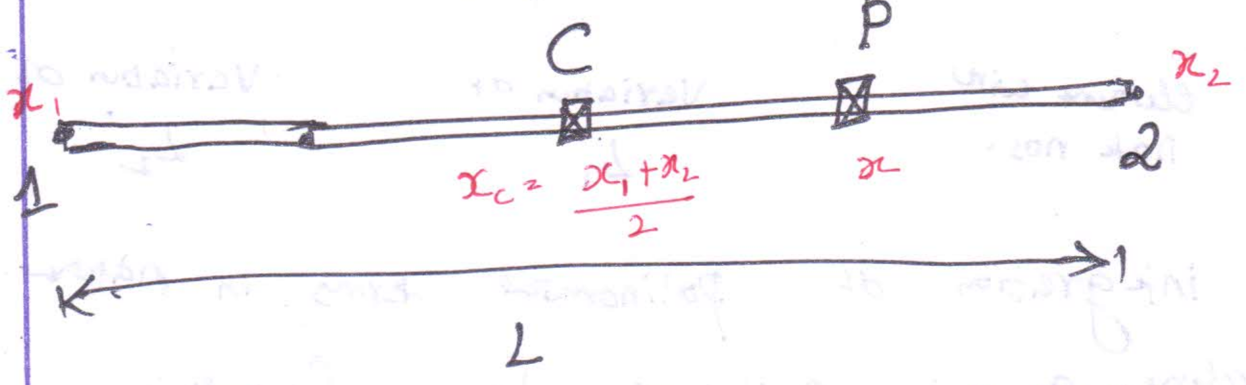
Natural co-ordinate; ξ

in 1D 'ξ' type Natural co-ordinate is also used.



Natural Co-ordinate 'ξ'

in 1D problems, the 'ξ' type natural co-ordinate is also used.



1 & 2 → Nodes, x_1, x_2 → Coordinates

C → Centre node of node 1 & 2; P is the point reffer...

The natural co-ordinate 'ξ' for any point in the element is defined as.

$$\xi = \frac{PC}{\left(\frac{x_2 - x_1}{2}\right)} = \left(\frac{PC}{l/2}\right) \Leftarrow (x_2 - x_1 = l)$$

$$= \frac{2}{l} * PC = \frac{2}{l}(x - x_c)$$

$$\xi = \frac{2}{l}(x - x_c)$$

$$\left(PC \Rightarrow x - x_c \right)$$

$$= \left(x - \frac{(x_1 + x_2)}{2}\right)$$

$$= \frac{2}{l}\left[x - \frac{(x_1 + x_2)}{2}\right]$$

$$= \frac{2}{l}\left[x - \left(\frac{x_2 + x_1}{2}\right)\right]$$

adding + subs x_1

$$= \frac{2}{l}\left[x - \left(\frac{x_2 + x_1 + x_1 - x_1}{2}\right)\right]$$

$$= \frac{2}{l}\left[x - \left(\frac{x_2 - x_1 + 2x_1}{2}\right)\right]$$

$$\xi = \frac{2}{l}\left[x - \left(l/2 + x_1\right)\right] \qquad (x - x_1 = l)$$

$$\frac{\xi l}{2} = x - l/2 - x_1$$

$$\boxed{(\xi + 1)\frac{l}{2} = x - x_1}$$

Now applying the boundary cond

$x = x_1$ @ node = 1

$$l/2\,(\xi + 1) = 0$$

$$\xi = -1$$

$x = x_2$ at node 2

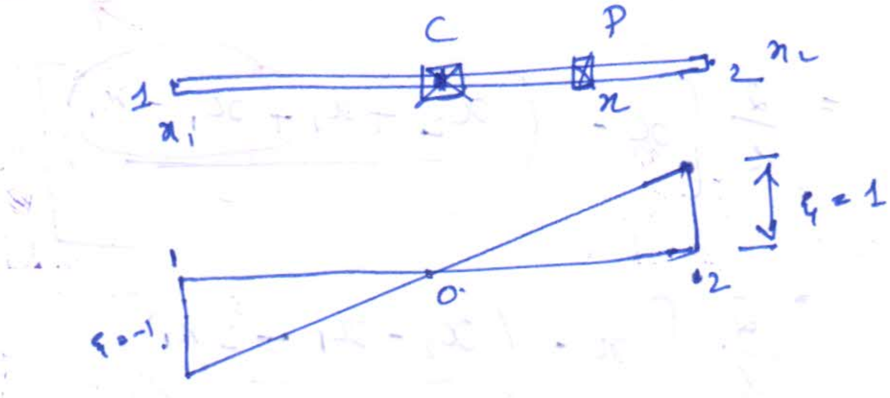
$$l/2\,(\xi + 1) = x_2 - x_1$$

$$\cancel{l}/2\,(\xi + 1) = \cancel{l}$$

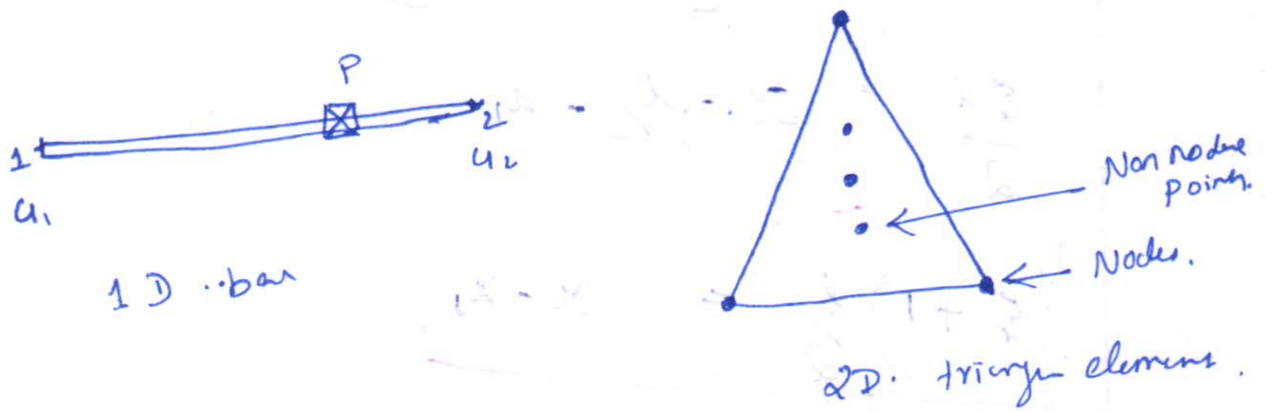
$$\xi = 2 - 1$$

$$\xi = 1$$

Variation of Natu Co-ord ξ.



Shape functions:



The Values of the field variable computed at the nodes are used to approximate the values at non-nodal points by interpolation of the nodal values.

eg. for triangle element :

* Nodes are exterior at any point within the elements. the field variable is described by the following approximate relation.